Chaos Theory, Edward Lorenz, and *Deterministic Nonperiodic Flow*

Greg Herman

Henri Poincaré

Born 29 April 1854 in Nancy, France to an affluent family

- Gifted student in almost all subjects as a child
- 1870: Served on Ambulance Corps during Franco-Prussian War
- 1873-1879: Studied at École Polytechnique (1873-1875) , and later at the University of Paris, under advisor Charles Hermite
- Simultaneously studied (1875-79) mining engineering at École des Mines, and subsequently joined the Corps des Mines

Poincaré

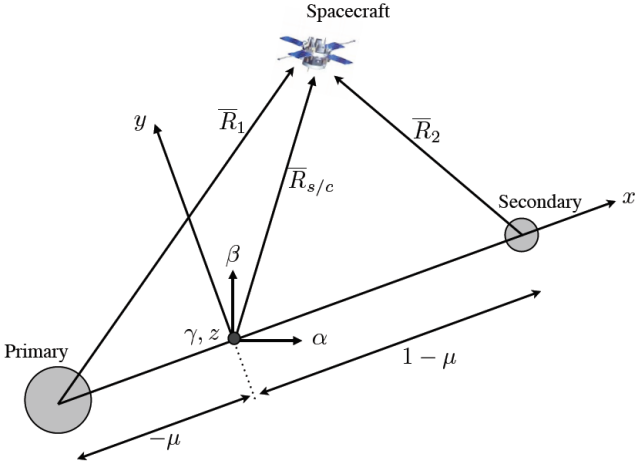
- Worked primarily as a professor of mathematics (University of Caen, University of Paris, École Polytechnique), becoming highly respected in the field
- Continued work as a mining engineer too, becoming chief engineer in 1893 and inspector general in 1910
- Contributions to many different areas of mathematics, mathematical physics, and celestial mechanics

Poincaré- Contributions

- Qualitative Theory of Differential Equations
- Preliminary Theory of Special Relativity
- Founding field of Algebraic Topology; significant advancements in Non-Euclidean Geometry
- Poincaré Recurrence Theorem
- Poincaré Conjecture
  - Every simply connected, closed 3-manifold is homeomorphic to the 3-sphere
  - Extension of what is known and proven for 2-manifolds (all that are closed and simply connected are homeomorphic to the sphere)
  - Critical interest to mathematics, unsolved for a century
  - One of the seven Millennium Prize Problems, and the only one that's been solved
- Three Body Problem
- Several others…

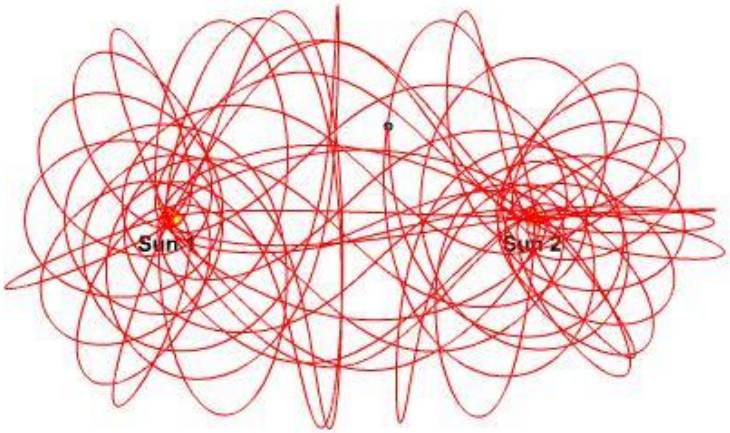
Three (n) Body Problem

- Given objects of masses of m_1, m_2, …, m_n with respective initial positions $r_i=(x_i, y_i, z_i)$, considering only Newton's Law of Universal Gravitation and neglecting all other forces, describe the position of each body as a function of time $r_i(t)$
- Two-body problem (n=2) solved by Bernoulli in 1734
- Solution to Three-body problem of great interest; king of Sweden offered prize for solution to the problem in 1887
- Poincaré worked on this problem extensively; did not find a general solution, but made extensive progress on a restricted version of the problem
  - Assumed third mass negligible compared to the other two
  - Assumed masses one and two in circular orbi about their collective center of mass



Three Body Problem: Cont'd

- In solving this *Restricted Circular Three-Body Problem*, Poincaré discovered the solution of the third mass's position (having put the other two in a reference frame that remains on a fixed axis for all time) is nonperiodic, and nonasymptotic
- Solutions were also highly sensitive to the initial conditions- small displacements of the third body led to large changes in its trajectory
- Karl Sundman found a complete solution for the n=3 case in 1912, the year of Poincaré's passing
- Solution to the general case found in the late 20th century



Birth of Chaos Theory

- Poincaré is often credited to being the first significant advocate of Chaos Theory, in association with his work on the Three-Body Problem
- Argued linear theory insufficient to explain observed behavior in many dynamical systems, including the sensitivity to initial conditions seen in the 3-body problem

Edward Lorenz: Early Years

- Born 23 May 1917 in West Hartford, CT
- Studied Mathematics at Dartmouth and Harvard
  - B.A. 1938, A.M. 1940
- Served as meteorologist for the Air Force during World War II (1942-1946)
- Upset about lives and equipment lost due to poor forecasts, decided to pursue meteorology upon returning
  - S.M. Meteorology 1943 (MIT)
  - ScD Meteorology 1948 (MIT)

Edward Lorenz: 1948-1961

- Meteorologist at MIT 1948-54
- Became Assistant Professor in 1954
- Early work focused primarily on fluid dynamics and general circulation.
- In later half of 1950s, began focusing more on NWP
  - Studied both dynamic forecasting based on Rossby's work, in addition to statistical forecasting

To some meteorologists the statistical method has appeared to be the antithesis of the dynamical method, since, although both methods are objective, the former seems to disregard the dynamics of the atmosphere, while the latter seems to disregard the statistics. It should be mentioned, then, that the dynamic method as practiced is not entirely free of empirical relations, for the dynamic equations are not ordinarily integrated in their primitive form, and the modifications made, such as the geostrophic approximation and the neglect of the vertically integrated divergence, are suggested by the observed behavior of the atmosphere rather than by pure dynamic theory. Likewise, in statistical studies the choice of predictors is often based upon dynamic considerations.

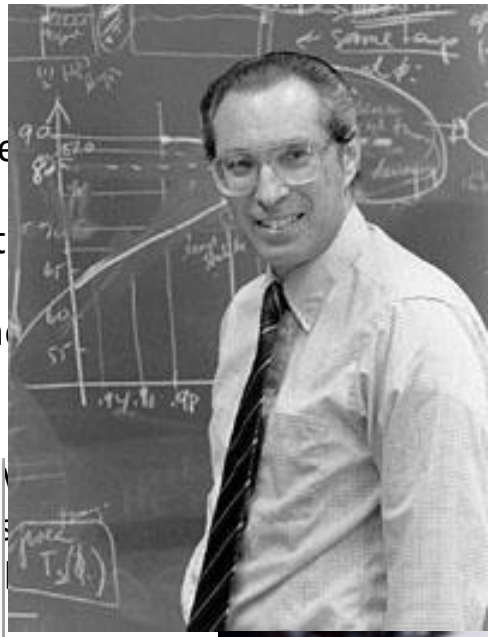
Edward Lorenz: 1960-1963

- Studying system of twelve equations modeling the atmosphere
- Printed out the variable values at each time step of the simulation
- Decided to investigate the system some more, wanted to run out the simulation longer
- Expensive to run at the time; decided to save computational time by starting at a mid-point in time in the model simulation using the printout
- Restarted the model, and expecting the results to be identical for the first 15-30 minutes, left the room for 30 minutes
- When he came back…

Edward Lorenz: 1960-1963

- …the solutions were radically different!
- Perplexed at how this could happen. The computer model, after all, is necessarily deterministic, even if what it's simulating is not
- Ed Lorenz, friend and colleague Barry Saltzman, and Lorenz's graduate student, Ellen Fetter, perused the data trying to figure out an explanation
- Fetter noticed that the printout had truncated the variable printed on the right side of the sheet
  - Prepared simulation with six decimal places of precision, only three on the printout
- Printer, lacking sufficient space, had simply truncated/deleted the last few columns!
- Lorenz had restarted the simulation with the variable as 0.506- what was on the printout- but, in reality, what the computer had stored in memory at that time step in the first simulation was 0.506127
- Lorenz was skeptical that such a small change could yield the huge differences he was seeing.
- Saltzman, less skeptical, had recently been working on convection equations, and had noted nonperiodicity in the solutions
- Investigating the matter further, this led Lorenz to his famous paper: *Deterministic Nonperiodic Flow*

Edward Lorenz: 1960-1963



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Deterministic Nonperiodic Flow[1]

EDWARD N. LORENZ

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ABSTRACT

Finite systems of deterministic ordinary nonlinear differential equations may be designed to represent forced dissipative hydrodynamic flow. Solutions of these equations can be identified with trajectories in phase space. For those systems with bounded solutions, it is found that nonperiodic solutions are ordinarily unstable with respect to small modifications, so that slightly differing initial states can evolve into considerably different states. Systems with bounded solutions are shown to possess bounded numerical solutions.

A simple system representing cellular convection is solved numerically. All of the solutions are found to be unstable, and almost all of them are nonperiodic.

The feasibility of very-long-range weather prediction is examined in the light of these results.

1. Introduction

Thus there are occasions when more than the statistics of irregular flow are of very real concern.

Lorenz '63: Definitions- Preliminaries

- Phase Space:
  - For any dynamical system, some number M of model variables can uniquely specify the system state.
  - This can be expressed as a vector or point of the form: x=$(m_1, m_2, \ldots, m_M)$
  - Each variable can be plotted on a separate dimension in an M-dimensional space termed *phase space*
  - Idea of phase space not new; often attributed to Liouville (1838)
- Trajectory:
  - At any time, the state of an evolving system may be plotted as a point x(t).
  - This set of points over all time forms a traversal or path through phase space, known as a *trajectory*

Lorenz '63: Preliminaries

- Atmosphere is a forced, dissipative system
- Thus interested in studying this class of systems

Forced dissipative systems of this sort are typified by the system

$$dX_i/dt = \sum_{j,k} a_{ijk} X_j X_k - \sum_j b_{ij} X_j + c_i, \tag{4}$$

where $\sum a_{ijk} X_i X_j X_k$ vanishes identically, $\sum b_{ij} X_i X_j$ is positive definite, and $c_1, \cdots, c_M$ are constants. If

$$Q = \tfrac{1}{2} \sum X_i^2, \tag{5}$$

and if $e_1, \cdots, e_M$ are the roots of the equations

$$\sum_j (b_{ij} + b_{ji}) e_j = c_i, \tag{6}$$

Lorenz '63: Forced Dissipative Systems

$$\frac{dQ}{dt} = \frac{d}{dt}\frac{1}{2}\sum_i {X_i}^2 = \frac{1}{2}\sum_i \frac{d}{dt}{X_i}^2 = \frac{1}{2}2\sum_i X_i\frac{dX_i}{dt} = \sum_i X_i\frac{dX_i}{dt}$$

$$= \sum_i X_i\left(\sum_j\sum_k a_{ijk}X_jX_k - \sum_j b_{ij}X_j + c_i\right)$$

$$= \sum_i\sum_j\sum_k a_{ijk}X_iX_jX_k - \sum_i\sum_j b_{ij}X_iX_j + \sum_i X_i\left(\sum_j\left(b_{ij} + b_{ji}\right)e_j\right)$$

$$= \sum_i\sum_j b_{ij}X_iX_j + b_{ij}X_ie_j + b_{ij}X_je_i + \left(b_{ij}e_ie_j - b_{ij}e_ie_j\right)$$

$$= \sum_i\sum_j b_{ij}e_ie_j - \sum_i\sum_j b_{ij}(X_i - e_i)\left(X_j - e_j\right)$$

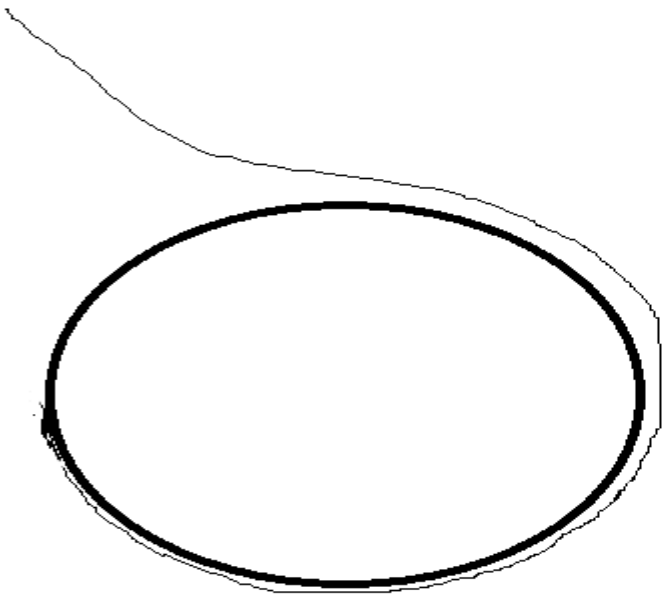
- dQ/dt vanishes on the surface of an ellipsoid E- positive within it, negative outside it
- Q constant on spherical surfaces
- All trajectories in a forced dissipative system will be thus confined to the interior of a sphere R which encloses E
- All trajectories in a forced dissipative system are thus said to be *bounded*

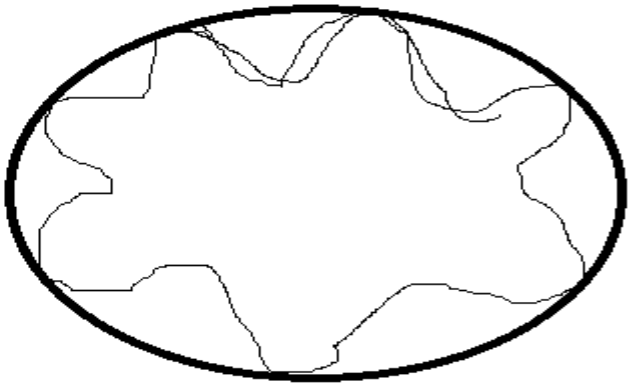
Lorenz '63: Definitions- Preliminaries

- Limit Point: A limit point of a trajectory P is a point that is approached arbitrarily close to a point P_0 arbitrarily often. There exist infinitely many times t that $||P(t)-P_0|| < \varepsilon$ for any small positive ε.
- Limit Trajectory(ies): The set of limiting points of a trajectory P

Lorenz ’63: Definitions- Transient Properties

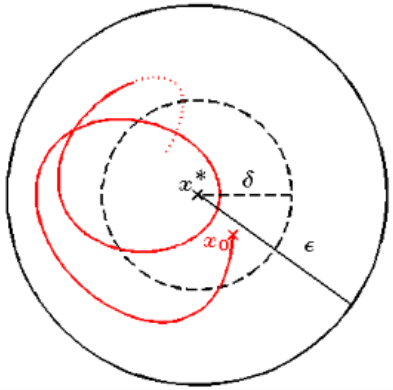
- Central: A trajectory P is called *central* if, for all time, P remains confined within its own limiting trajectories
- Noncentral: Any trajectory that is not central





Lorenz '63: Definitions- Stability

- Stable at a point: A trajectory P is termed *stable at a point* P_0 if any trajectory passing within a distance $\delta(t_0,\varepsilon) > 0$ of P_0 at time t_0 will remain within a distance ε for all $t > t_0$ and $\varepsilon > 0$
- Stable trajectory: A trajectory that is stable at a point (stable at one point implies stable at all points)
  - A trajectory Q that approximates P at time t_0 will continue to do so for all time.
- Unstable: Any trajectory that is not stable
- Uniformly Stable: A trajectory P is termed *uniformly stable at a point* P_0 if any trajectory passing within a distance $\delta(\varepsilon) > 0$ of P_0 at time t_0 will remain within a distance $\varepsilon > 0$ for all $t > t_0$
  - δ independent of time; if a trajectory *ever* goes within δ of P_0 at any time, it will remain within the ε bound for all time



Lorenz '63: Definitions- Periodicity

- Periodic: A trajectory P is *periodic* if, for some time interval τ, $P(t) = P(t+\tau)$ for all t
  - E.g. Loop
- Quasi-Periodic: A trajectory P is *quasi-periodic* if there exists a time interval τ such that for all times t, $||P(t+\tau) - P(t)|| < \varepsilon$ for some arbitrarily small $\varepsilon > 0$
  - E.g. Some traverses on the surface of a torus
- Nonperiodic: Any trajectory P that is not quasi-periodic.
  - Note: $||P(t+\tau) - P(t)||$ may get arbitrarily small for some t and τ, but cannot hold for all t (as $t \to \infty$)

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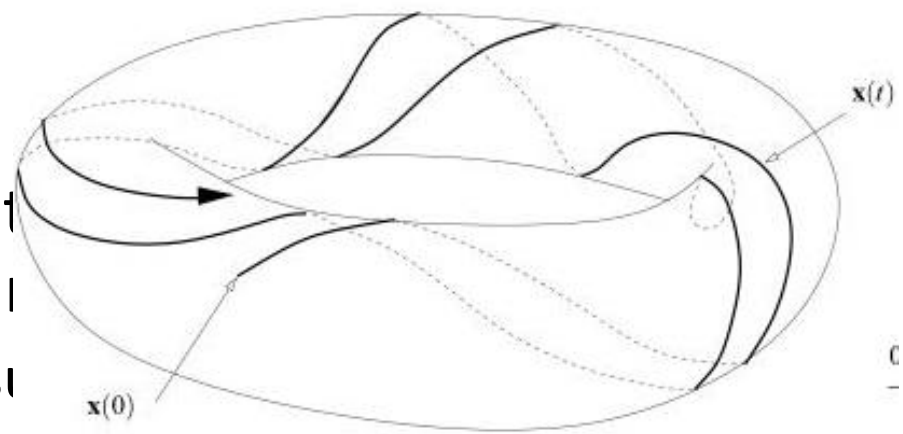

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Lorenz ‘63: Booleans

- P: Periodic
- QP: Quasi-Periodic
- ¬P/¬QP: Nonperiodic
- C/¬C: Central/Noncentral
- S/¬S: Stable/Unstable
- US: Uniformly Stable
- P→QP (by definition)
- US→S (by definition)

Lorenz '63: Claims

- P→C

Periodic trajectories are obviously central.

- S(LT)→QP

We can now establish the theorem that a trajectory with a stable limiting trajectory is quasi-periodic.

- S^C→QP; ¬P^C → ¬S

It follows immediately that a stable central trajectory is quasi-periodic, or, equivalently, that a nonperiodic central trajectory is unstable.

- US^ ¬C→QP; ¬P^ ¬C→ ¬US

As for noncentral trajectories, it follows that a uniformly stable noncentral trajectory is quasi-periodic, or, equivalently, a nonperiodic noncentral trajectory is not uniformly stable.

  – Essentially makes long-range daily weather forecasting impossible to do accurately

Lorenz '63: Claims Summary

In summary, we have shown that, subject to the conditions of uniqueness, continuity, and boundedness prescribed at the beginning of this section, a central trajectory, which in a certain sense is free of transient properties, is unstable if it is nonperiodic. A noncentral trajectory, which is characterized by transient properties, is not uniformly stable if it is nonperiodic, and, if it is stable at all, its very stability is one of its transient properties, which tends to die out as time progresses. In view of the impossibility of measuring initial conditions precisely, and thereby distinguishing between a central trajectory and a nearby noncentral trajectory, all nonperiodic trajectories are effectively unstable from the point of view of practical prediction.

Lorenz ’63: The System- Setup

In this section we shall introduce a system of three ordinary differential equations whose solutions afford the simplest example of deterministic nonperiodic flow of which the writer is aware.

- Simplification of the finite amplitude convection equations of Saltzman (1962)
- Fluid of uniform depth H; upper and lower boundaries experience constant temperature difference ΔT

Lorenz '63: The Equations

$$\frac{dX}{d\tau} = -\sigma X + \sigma Y$$

$$\frac{dY}{d\tau} = -XZ + rX - Y$$

$$\frac{dZ}{d\tau} = XY - bZ$$

$$\sigma = \frac{\nu}{\kappa}; \mathrm{r} = \frac{R_a}{R_c}; b = \frac{4}{1 + a^2}$$

$$R_a = \frac{g\alpha H^3 \Delta T}{\kappa\nu};\ R_c = \frac{\pi^4 (1 + a^2)^3}{a^2}$$

$$\tau = \left(\frac{\pi}{H}\right)^2 (1 + a^2)\kappa \mathrm{t}$$

Lorenz ‘63: The Terms

H: Fluid Depth

a: Length Scale

τ: Dimensionless Time

ν: Kinematic Viscosity

κ: Thermal Conductivity

R_a: Rayleigh Number

R_c: Critical Rayleigh Number

High $\frac{R_a}{R_c}$ →heat transfer by convection

Low $\frac{R_a}{R_c}$ →heat transfer by conduction

Lorenz '63: The Terms

- X ∝ Convection Intensity
- Y ∝ $T_{asc} - T_{desc}$
- Z ∝ non-linearity of $\frac{dT}{dz}$
  - Z > 0 -> largest $\frac{dT}{dz}$ near boundaries
- Point P(t) in phase space defined by (X(t),Y(t),Z(t))

Equations may give realistic results when the Rayleigh number is slightly supercritical, but their solutions cannot be expected to resemble those of Saltzman when strong convection occurs, in view of the extreme truncation.

Lorenz '63: Relating the System to the Theory

- Linearized Equations:
- $$\begin{bmatrix} \frac{dx}{d\tau} \\ \frac{dy}{d\tau} \\ \frac{dz}{d\tau} \end{bmatrix} = \begin{bmatrix} -\sigma & \sigma & 0 \\ r - Z & -1 & -X \\ Y & X & -b \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$
- It may be readily shown that, for a square matrix A:

$$\det(e^{A}) = e^{Tr(A)}$$
$$\det(e^{tA}) = e^{Tr(tA)} = 1 + Tr(tA) + O(t^2) \approx tTr(A)$$

- A system of the form p'=Ap, like the linearized system above, has a solution: $p(t) = p(0)e^{At}$
- From this, it follows immediately that for some initial volume V, the volume change $\frac{dV}{d\tau}$ in a linearized system of this form over a small time increment $d\tau$ may be expressed as:

$$\frac{dV}{d\tau} = \mathrm{V} * \det(e^{A\tau}) = VTr(A) = -V(\sigma + 1 + b)$$

Lorenz ’63: Relating the System to the Theory

- From $\frac{dV}{d\tau} = -V(\sigma + 1 + b)$, it is apparent that a volume V in phase space will shrink at a rate independent of the system variables
- Thus as $\tau \to \infty, V \to 0$
- It then follows that all trajectories in the Lorenz ‘63 system are eventually confined to a subspace spanning zero volume at large time
- This necessitates that all trajectories in the Lorenz system be *central*

Lorenz '63: Solutions

$$\frac{dX}{d\tau} = -\sigma X + \sigma Y$$
$$\frac{dY}{d\tau} = -XZ + rX - Y$$
$$\frac{dZ}{d\tau} = XY - bZ$$

- Steady-State Solution: X=Y=Z=0 (trivial solution)
- Characteristic Equation: $\det(A - \lambda I) = 0$

$$\left|\begin{bmatrix} -\sigma & \sigma & 0 \\ r - Z & -1 & -X \\ Y & X & -b \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right| = \begin{vmatrix} -\sigma - \lambda & \sigma & 0 \\ r & -1 - \lambda & 0 \\ 0 & 0 & -b - \lambda \end{vmatrix}$$
$$= (\sigma + \lambda)\begin{vmatrix} 1 + \lambda & 0 \\ 0 & b + \lambda \end{vmatrix} - \sigma \begin{vmatrix} r & 0 \\ 0 & -(b + \lambda) \end{vmatrix}$$
$$= (b + \lambda)(\lambda^2 + (\sigma + 1)\lambda + \sigma(1 - r)) = 0$$

- Roots: 1) $\lambda = -b$
- 2) $\lambda = -\frac{1+\sigma}{2} - \frac{1}{2}\sqrt{(1 + \sigma)^2 - 4\sigma(1 - r)}$
- 3) $\lambda = -\frac{1+\sigma}{2} + \frac{1}{2}\sqrt{(1 + \sigma)^2 - 4\sigma(1 - r)}$
- (1) and (2) always negative, (3) positive if r > 1
- ∴ Threshold for onset of convection is r=1, as suggested by Rayleigh

Lorenz '63: Solutions

$$\frac{dX}{d\tau} = -\sigma X + \sigma Y$$
$$\frac{dY}{d\tau} = -XZ + rX - Y$$
$$\frac{dZ}{d\tau} = XY - bZ$$

- Two other steady-state solutions if r > 1:

1. $X = Y = \sqrt{b(r-1)},\ Z = r - 1$
2. $X = Y = -\sqrt{b(r-1)}, Z = r - 1$

- Characteristic Equation:

$$\lambda^3 + (\sigma + b + 1)\lambda^2 + (r + \sigma)b\lambda + 2\sigma b(r-1) = 0$$

$$\therefore \begin{cases} (\sigma < b+1) \rightarrow steady\ convection\ always\ stable \\ (\sigma > b+1) \rightarrow \begin{cases} unstable \quad r > \dfrac{\sigma(\sigma+b+3)}{\sigma-(b+1)} \\ stable\ r < \dfrac{\sigma(\sigma+b+3)}{\sigma-(b+1)} \end{cases} \end{cases}$$

- Complex roots → convection intensity will oscillate

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- Char

The presence of complex roots of (34) shows that if unstable steady convection is disturbed, the motion will oscillate in intensity. What happens when the disturbances become large is not revealed by linear theory. To investigate finite-amplitude convection, and to study the subspace to which trajectories are ultimately confined, we turn to numerical integration.

$$stable\ r < \frac{\quad}{\sigma - (b+1)}$$

- Complex roots → convection intensity will oscillate

Lorenz '63: Numerical Integration

- System parameterized by σ, a; must be specified.
- Let $\sigma = 10; a^2 = \frac{1}{2} \rightarrow b = \frac{8}{3} \rightarrow r_{crit} = 24.74$
- Let r=28 (supercritical)
- Non-Trivial Convection Steady States:

$$\left(6\sqrt{2}, 6\sqrt{2}, 27\right), \left(-6\sqrt{2}, -6\sqrt{2}, 27\right)$$

- Initialized at (0,1,0)

Lorenz ’63: Numerical Integration

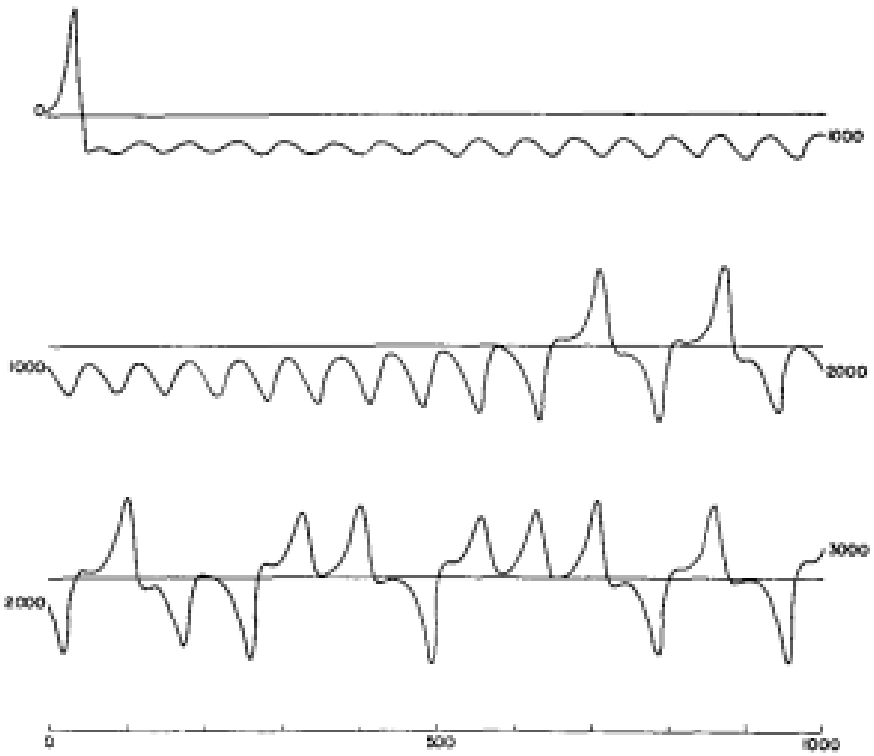
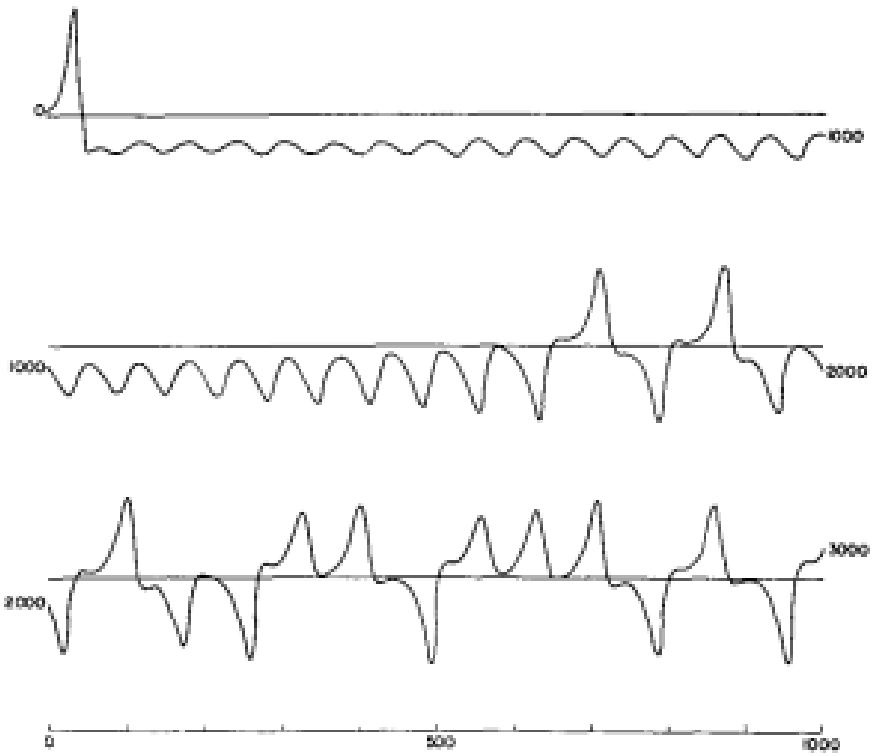


FIG. 1. Numerical solution of the convection equations. Graph of Y as a function of time for the first 1000 iterations (upper curve), second 1000 iterations (middle curve), and third 1000 iterations (lower curve).

Lorenz ’63: Numer



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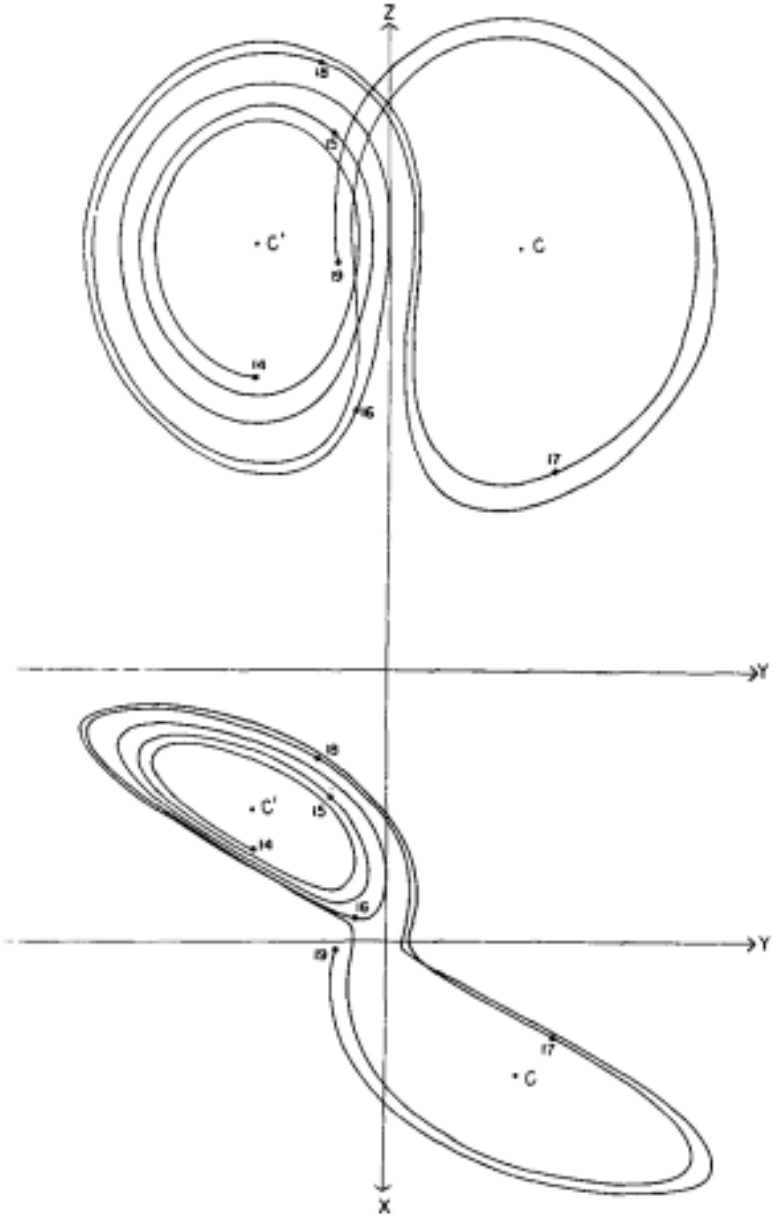


FIG. 2. Numerical solution of the convection equations. Projections on the X-Y-plane and the Y-Z-plane in phase space of the segment of the trajectory extending from iteration 1400 to iteration 1900. Numerals "14," "15," etc., denote positions at iterations 1400, 1500, etc. States of steady convection are denoted by C and C'.

Lorenz '63: Numerical Integration

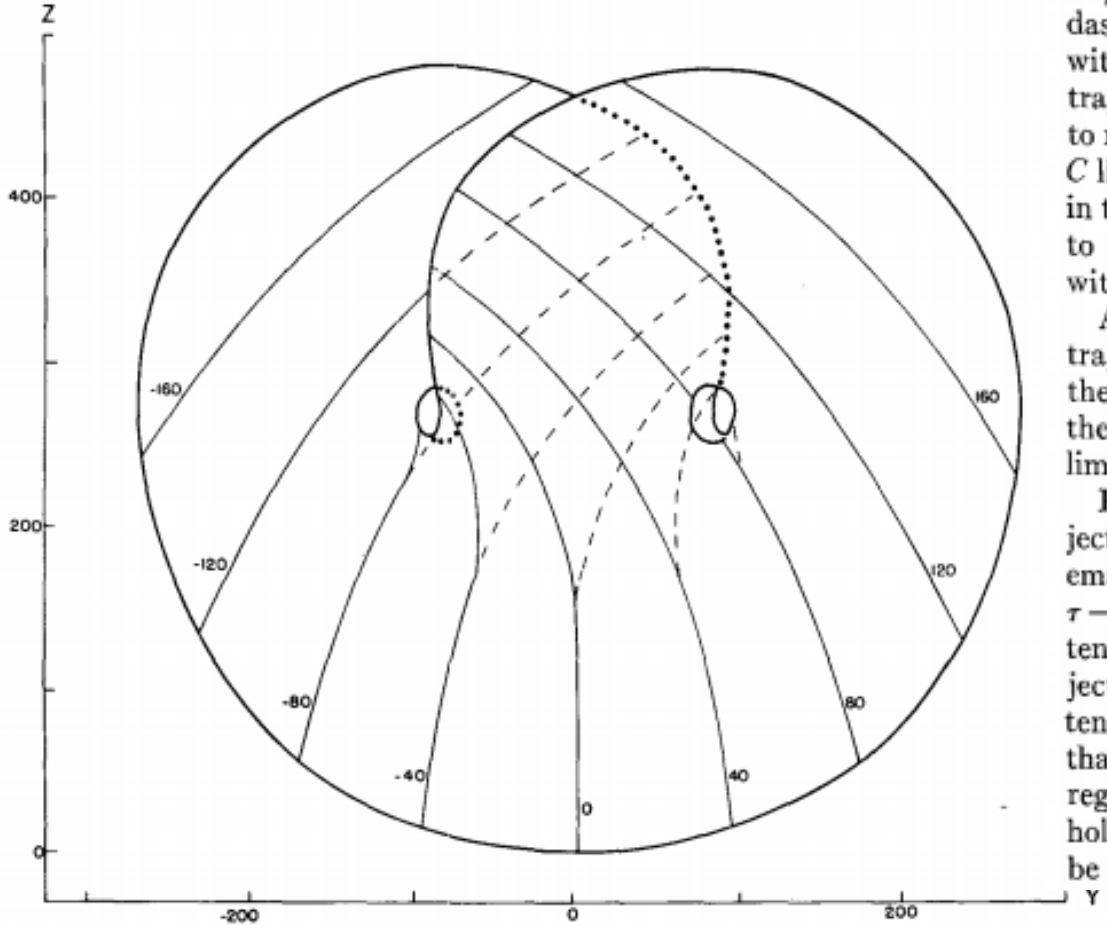
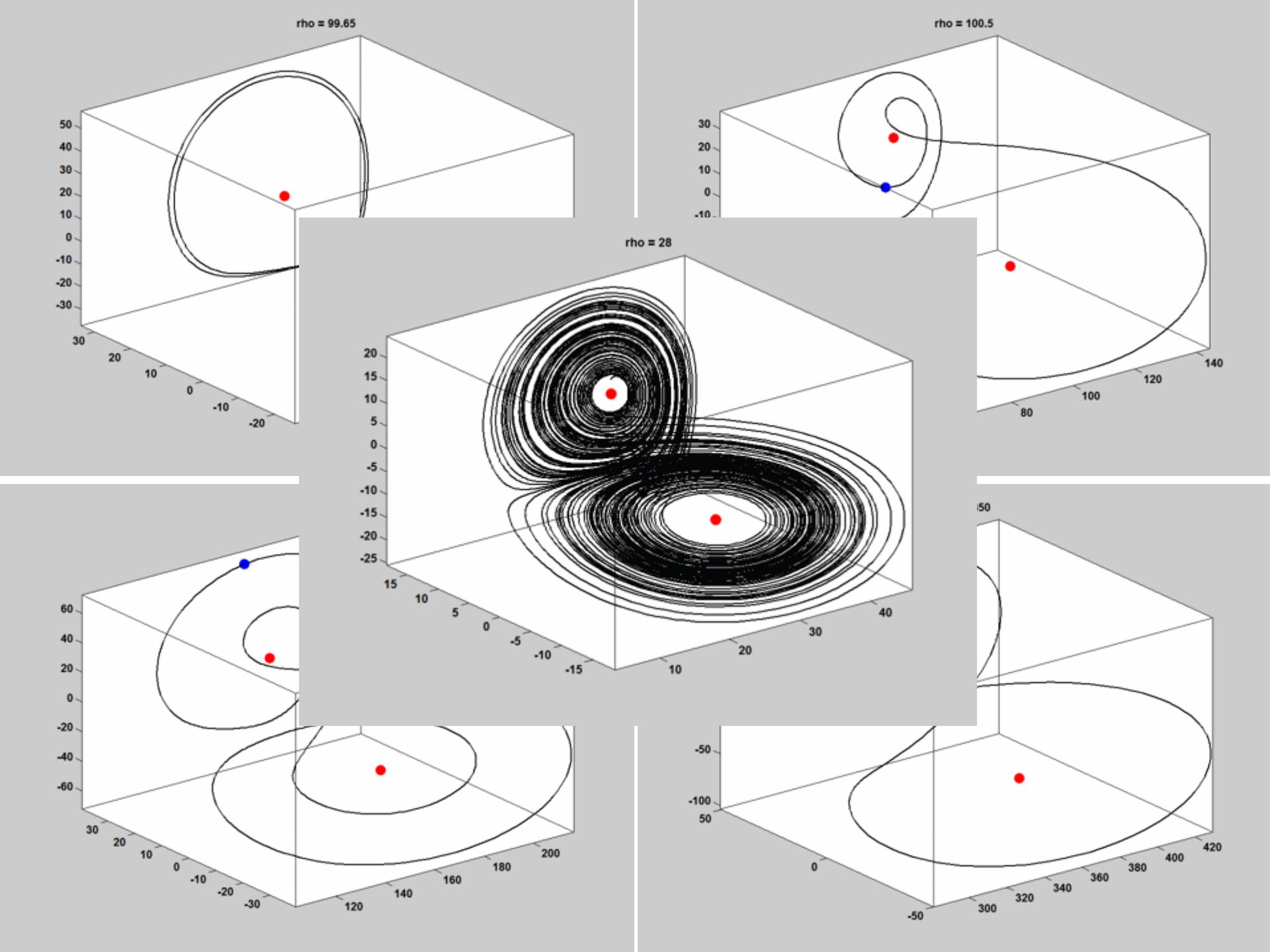


FIG. 3. Isopleths of X as a function of Y and Z (thin solid curves), and isopleths of the lower of two values of X, where two values occur (dashed curves), for approximate surfaces formed by all points on limiting trajectories. Heavy solid curve, and extensions as dotted curves, indicate natural boundaries of surfaces.

In Fig. 3 the thin solid lines are isopleths of X, and where two values of X exist, the dashed lines are isopleths of the lower value. Thus, within the limits of accuracy of the printed values, the trajectory is confined to a pair of surfaces which appear to merge in the lower portion of Fig. 3. The spiral about C lies in the upper surface, while the spiral about C' lies in the lower surface. Thus it is possible for the trajectory to pass back and forth from one spiral to the other without intersecting itself.

Additional numerical solutions indicate that other trajectories, originating at points well removed from these surfaces, soon meet these surfaces. The surfaces therefore appear to be composed of all points lying on limiting trajectories.

Because the origin represents a steady state, no trajectory can pass through it. However, two trajectories emanate from it, i.e., approach it asymptotically as $\tau \rightarrow -\infty$. The heavy solid curve in Fig. 3, and its extensions as dotted curves, are formed by these two trajectories. Trajectories passing close to the origin will tend to follow the heavy curve, but will not cross it, so that the heavy curve forms a natural boundary to the region which a trajectory can ultimately occupy. The holes near C and C' also represent regions which cannot be occupied after they have once been abandoned.


rho = 99.65
rho = 100.5
rho = 28

Lorenz ‘63: Characteristics of Trajectories

- All trajectories in the Lorenz System are *central*
  - Proven both by linear theory and numerical integration
- All trajectories in the Lorenz System are *unstable*
- Some trajectories in the Lorenz System are *periodic;* some are *nonperiodic*
- No trajectory in the Lorenz System is *quasi-periodic*

Lorenz ‘63: Atmospheric Implications

When our results concerning the instability of nonperiodic flow are applied to the atmosphere, which is ostensibly nonperiodic, they indicate that prediction of the sufficiently distant future is impossible by any method, unless the present conditions are known exactly. In view of the inevitable inaccuracy and incompleteness of weather observations, precise very-long-range forecasting would seem to be non-existent.

Lorenz '63: Atmospheric Implications

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Fri 12/4/2015	48°	25°	0.15 in	0 in	Mostly cloudy with some sleet	48°	19°
Sat 12/5/2015	44°	24°	0.13 in	0 in	Sun and some clouds	48°	19°
Sun 12/6/2015	40°	23°	0.14 in	0.6 in	Spotty showers	48°	19°
Mon 12/7/2015	39°	16°	0.02 in	0.1 in	Cloudy, a rain or snow shower	47°	19°
Tue 12/8/2015	40°	23°	0 in	0 in	Sunny	47°	18°
Wed 12/9/2015	42°	12°	0.09 in	1.9 in	Sunny	47°	18°
Thu 12/10/2015	38°	11°	0.06 in	0.9 in	Morning snow, then a flurry	47°	18°
Fri 12/11/2015	39°	17°	0 in	0 in	Sunny	47°	18°
Sat 12/12/2015	42°	21°	0 in	0 in	Sunshine	46°	18°
Sun 12/13/2015	43°	21°	0 in	0 in	Sunshine	46°	18°
Mon 12/14/2015	43°	22°	0 in	0 in	Sunshine	46°	18°
Tue 12/15/2015	47°	23°	0 in	0 in	Abundant sunshine	46°	18°
Wed 12/16/2015	42°	23°	0.09 in	0.1 in	A little afternoon rain	46°	18°
Thu 12/17/2015	38°	20°	0.02 in	0.2 in	Variably cloudy, snow showers	46°	18°

Lorenz '63: Atmospheric Implications

If it is true that two analogues have occurred since atmospheric observation first began, it follows, since the atmosphere has not been observed to be periodic, that the successions of states following these analogues must eventually have differed, and no forecasting scheme could have given correct results both times. If, instead, analogues have not occurred during this period, some accurate very-long-range prediction scheme, using observations at present available, may exist. But, if it does exist, the atmosphere will acquire a quasi-periodic behavior, never to be lost, once an analogue occurs. This quasi-periodic behavior need not be established, though, even if very-long-range forecasting is feasible, if the variety of possible atmospheric states is so immense that analogues need never occur. It should be noted that these conclusions do not depend upon whether or not the atmosphere is deterministic.

Lorenz ‘63: Aftermath

- Did NOT immediately catch on
  - Lorenz humble and soft-spoken
  - Three citations outside meteorology 1963-1973
- Eventually convinced to popularize his work in the 1970s and 1980s
  - Coined the term “Butterfly Effect”
  - Became especially well known after publication of James Gleick’s 1987 book *Chaos: Making a New Science*
  - Now considered one of the most impactful papers of the 20th century; >15,000 citations today
  - Lorenz’s ‘63 system the basis for countless subsequent studies on chaos and predictability

Lorenz ‘63: Aftermath

- Did NOT immediately catch on
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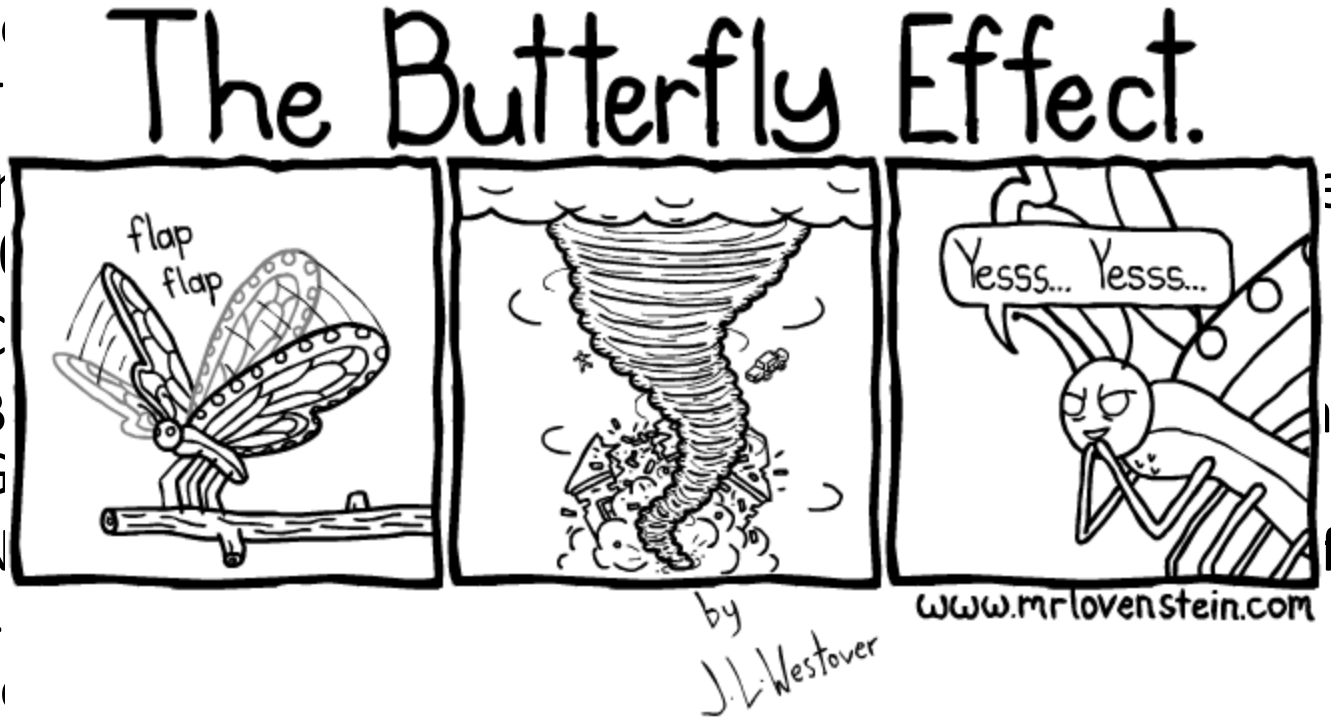

Lorenz ‘63: Aftermath

- Changes in Meteorology
  - Enhanced effort to improve model initializations
  - Enhanced effort at improved modeling
  - Ensemble Forecasting

Lorenz ‘63: Aftermath

- Awarded many accolades for his career of work, especially on chaos and predictability
  - Rossby Medal, 1969
  - Gold Medal, Royal Meteorological Society, 1973
  - Fellow, National Academy of Sciences, 1975
  - Kyoto Prize, 1991
- Retired to Emeritus status in 1987, but remained rather active in the field
- Died at 90 on 16 April, 2008 in Cambridge, MA
  - Apparently completed a manuscript with a co-author less than two weeks before his passing

QUESTIONS?